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ON THE AUTOMATIC REGULATION OF OUTPUT IN CENTRIFUGAL COMPRESSORS

By V. F. Ris

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ON THE AUTOMATIC REGULATION OF OUTPUT IN CENTRIFUGAL COMPRESSORS[1]

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SUMMARY

This paper discusses the theory and design of dynamic "pressure augmentors" (diaphragms equal orifice plates and nozzles) and various forms of "pressure multipliers" (simple venturi tubes, Rateau-type multiple venturis, and a combination of shaped nozzle and simple venturi developed by the author). No complete theory of pressure multiplication is yet available; conditions of governing are discussed in relation to pressure-augmenting devices fitted either on the suction or the pressure side of the blower; fluctuations of output and power consumption caused by the presence of an augmentor are analyzed with the result that fitting on the pressure side appears generally preferable. Some considerations on the suitable design and selection of pressure-augmenting devices are appended.

I. DEVICES FOR AUGMENTING THE GOVERNING FORCE

The problem of controlling the output of a centrifugal compressor obviously consists in maintaining a constant volume of discharge to the pipe line or air receiver; either by correspondingly controlling the blower speed or by throttling the intake volume at constant blower speed.

In both cases the regulating impulse is provided by indirect forces affecting the discharge volume: the dynamic pressure, any pressure difference, or the resistance of

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[1] Sovetskoe Kotloturbostroenie. No. 8, Aug. 1940, pp. 261-269.

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some rigid body inserted in the stream. However, owing to the usually low flow velocity in the pipe line, these indirect forces are insufficient to ensure the necessary sensitivity of the governor; consequently, special pressure-augmenting devices are necessary increasing the dynamic force of the discharge such as the contraction of the flow channel by crifice plates or shaped nozzles or some form of pressure-multiplying device.

1. Pressure Augmenting by Diaphragms

(Orifice Plates) or Nozzles

Diaphragms and nozzles are the simplest form of pressure augmentor. However, they commonly cause excessive residual pressure losses and for the purpose in view (output regulation) are useful only in special cases.

The magnitude of the pressure differential $\Delta P'$ provided by an augmentor depends not only on the form of the latter but also on the manner of its arrangement in the flow — that is, the design and arrangement of the pipes transmitting the pressure drop to the governor membrane. Two possible cases for a pressure-augmenting device are:

Case 1 (fig. 1a): The pressure differential $\Delta P'$ is the difference of the static pressures P_I and P_{II} at the upstream and downstream faces of the constricted flow; that is, assuming γ equals constant and disregarding losses, approximately:

$$\Delta P' = P_I - P_{II} = \gamma_1 \frac{C_2^2 - C_1^2}{2g}$$

The difference in pressure $P_I - P_{II}$ between the upstream and downstream faces of the constricted flow is a measure of the discharge volume and will be denoted by ΔP. Then, for the present case:

$$\Delta P' = \Delta P \qquad (1)$$

Case 2 (fig. 1b): The pressure differential $\Delta P'$ is the difference between the total pressure $P_1 + \gamma_1 \frac{C'_1{}^2}{2g}$

measured at a distance of 0.6D to 1.0D in advance of the constriction, and the static pressure P_{II} measured at the downstream face of the constriction (at the pipe wall immediately behind the diaphragm or nozzle). The expression for the pressure difference $\Delta P'$ is consequently[1]:

$$\Delta P' = \left(P_1 + \gamma_1 \frac{C_1'^2}{2g}\right) - P_{II} = \Delta P + \gamma_1 \frac{C_1^2}{2g} \qquad (2)$$

Since by the Bernoulli equation:

$$P_1 + \gamma_1 \frac{C_1'^2}{2g} = P_2 + \gamma_1 \frac{C_2^2}{2g}$$

and $P_2 = P_{II}$, substitute in equation (2):

$$\Delta P' = \gamma_1 \frac{C_2^2}{2g} \qquad (2')$$

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[1]The pressure P_I at the tube wall immediately in front of the orifice plate or nozzle is greater than the pressure P_1 at section 1 owing to gradual retardation of the flow along the tube wall up to the apex of the right angle (here the theoretical velocity is zero). Consequently,

$$P_1 = P_I - \varphi_1 \gamma_1 \frac{C_1^2}{2g}$$

where the mean velocity C_1 is connected with the velocity C'_1 before the pitot tube by the relation $C'_1 = K_c C_1$. Consequently,

$$P_1 + \gamma_1 \frac{C_1'^2}{2g} = P_1 + (K_c^2 - \varphi_1) \quad \gamma_1 \frac{C_1^2}{2g}$$

The coefficient $K_c = \frac{C'_1}{C_1}$ depends on Re (Reynolds number) the position of the pitot tube and the surface roughness in the pipe. The coefficient φ_1 is 0.45 for smooth pipes and 0.357 for rough pipes. If the pitot tube is situated in the axis of the pipe the value of $(K_c^2 - \varphi_1)$ approaches unity as assumed in the derivation of equation (2).

Consequently, in this case the pressure difference $\Delta P'$ providing the governing force is equal to the dynamic pressure $\gamma_1 \frac{C_2^2}{2g}$ at the minimum section behind the constriction. This pressure difference is greater than the pressure drop in case 1 by the amount of the dynamic pressure in front of the constriction.

In practice, a third scheme of arrangement of the pressure augmentor, according to figure 1c, is frequently found. The pressure differential is the same as for case 2 (fig. b), but is pulsating owing to the higher velocities on the downstream side of the constriction at the mouth of the pitot tube.

The connection between the pressure difference ΔP obtained in an arrangement after case 1 and the discharge volume Q_1 in cubic meters per second, related to the condition of the flow upstream of the diaphragm or nozzle, is expressed by the following equation:

$$Q_1 = \alpha \sigma \left(\frac{d}{D}\right)^2 F \sqrt{2g \frac{\Delta P}{\gamma_1}} \tag{3}$$

where

d diameter of orifice or nozzle

D diameter of pipe line, $F = \frac{\pi}{4} D^2$

α discharge coefficient depending on $d : D$

σ compressibility coefficient depending on $\frac{\Delta P}{P_1}$ and $\frac{d}{D}$

γ_1 density of medium before the orifice or nozzle

For an arrangement after case 1 (from now on C_1 equals mean velocity in the pipe line before the pressure augmentor) there is, by equations (1) and (3):

$$\Delta P' \doteq \Delta P = \frac{1}{\sigma^2 \alpha^2 \left(\frac{d}{D}\right)^4} \gamma_1 \frac{C_1^2}{2g}$$

and for an arrangement after case 2:

$$\Delta P' = \Delta P + \gamma_1 \frac{C_1^2}{2g} = \left[\frac{1}{\sigma^2 \ \alpha^2 \left(\frac{d}{D}\right)^4} + 1\right] \gamma_1 \frac{C_1^2}{2g}$$

Denoting the coefficient indicating the increment of the pressure difference $\Delta P'$ produced by the augmentor over the dynamic pressure in the pipe line $\gamma_1 \frac{C_1^2}{2g}$ by M (coefficient of augmentation) gives

(a) For the arrangement after case 1:

$$M = \frac{1}{\sigma^2 \ \alpha^2 \left(\frac{d}{D}\right)^4} \tag{4}$$

(b) For the arrangement aftor case 2:

$$M = \frac{1}{\sigma^2 \ \alpha^2 \left(\frac{d}{D}\right)^4} + 1 \tag{5}$$

Commence by disregarding the effect of compressibility - that is, assume $\sigma = 1$ - and denote the value of M for $\sigma = 1$ by M_o. It must be remembered that, for an arrangement after case 1,

$$M = \frac{1}{\sigma^2} M_o \tag{6}$$

The selection of the type of pressure-augmenting dovice will be governed almost entirely by the residual pressure losses δP, produced by the augmentor.

For orifice plates and nozzles this will be (reference 1, p. 285):

$$\delta P = \left[\frac{1}{\mu \left(\frac{d}{D}\right)^2} - 1\right]^2 \gamma_1 \frac{C_1^2}{2g} \tag{7}$$

where μ is the coefficient of contraction of the stream; for nozzles $\mu = 1.0$; whereas for orifices it can be found by the formula (reference 1, p. 284):

$$\mu = \frac{\alpha}{\sqrt{1 + 0.55\ \alpha^2 \left(\frac{d}{D}\right)^4}} \tag{8}$$

If the residual losses δP are represented by:

$$\delta P = \xi\ \Delta P' \tag{9}$$

then the coefficient of residual loss is determined by:

$$\xi = \frac{1}{M_o}\left[\frac{1}{\mu\left(\frac{d}{D}\right)^2} - 1\right]^2 \tag{10}$$

In figure 2 the most important factors in the above expression – the ratio $d : D$ and the residual loss coefficient ξ – are plotted for convenience in calculation as function of the coefficient of augmentation M_o. According to this:

(a) The residual loss coefficient ξ increases with increasing coefficient M_o – first quickly, then more slowly.

(b) The arrangement after case 1 is only less favorable than case 2 for low values of M_o; for high values of M_o the two arrangements are practically equivalent.

Figure 2 also shows that with M_o equals constant and the same arrangement of the augmentor the residual pressure losses for orifices and nozzles are the same. Therefore, orifices should be preferred to nozzles as the simple and cheaper arrangment. The only exception is if the flow is contaminated with tarry refuse or dust; in such case a tenacious crust forms more quickly on orifices than on nozzles, producing a rounded edge to the aperture of the orifice plate, affecting its efficiency, and consequently the value of M_o.

2. The Use of Venturi Tubes and Pressure Multipliers as Pressure-Augmenting Devices

A pressure multiplier (choke) is a coaxial arrangement of venturi tubes of different diameters - a multiple-venturi-tube arrangement. These were suggested by Rateau (fig. 3). The disadvantage of the Rateau choke (usually consisting of a triple venturi tube) is its considerable structural length, which may attain 4D to 6D and makes practical application difficult. The present writer has, therefore, suggested an alternative arrangement (shown on fig. 7) and consisting of an orifice plate with a coaxial venturi tube in the aperture, which has a far lower structural length of 0.75D to 1.0D.

Theoretical analysis of the pressure-multiplier principle will first be applied to a Rateau-type choke consisting of a double venturi tube (fig. 3a) and then extended to other equivalent arrangements. The following notation will be used:

$F_{e\ I}$ entry section of the small venturi I

$F_{k\ I}$ minimum section of the small venturi I

$F_{a\ I}$ discharge section of the small venturi I

$F_{e\ II}$ entry section of the large venturi II

$F_{k\ II}$ minimum section of the large venturi II

$F_{a\ II}$ discharge section of the large venturi II

To simplify calculation, it will be assumed that $F_{e\ II} = F_{a\ II} = F$, the last being equal to $(\pi/4)D^2$ equals cross section of the pipe line; furthermore, that

$$m_I = \frac{F_{a\ I}}{F_{k\ I}}$$

$$m_{II} = \frac{F_{a\ II}}{F_{k\ II}}$$

$$n = \frac{F_{k\ II}}{F_{a\ I}}$$

being the ratios of all sections to the cross section of the pipe line F (excluding the entry sections which will be determined later) by the following expressions,

$$F_{k\ II} = \frac{F}{m_{II}} \tag{11}$$

$$F_{a\ I} = \frac{F_{k\ II}}{n} = \frac{F}{n\ m_{II}} \tag{12}$$

$$F_{k\ I} = \frac{F_{a\ I}}{m_I} = \frac{F}{n\ m_I\ m_{II}} \tag{13}$$

Three consecutive equations for the discharge can now be set up: for the portion of the flow passing through venturi I, for the portion of the flow between venturis I and II, and for the total flow. Then,

$$Q_I = C_{e\ I}\ F_{e\ I} = C_{a\ I} - F_{a\ I} \tag{14}$$

$$C'_e\ (F - F_{e\ I}) = C'_k\ (F_{k\ II} - F_{a\ I}) \tag{15}$$

$$Q_I = C_1\ F = C_{a\ I}\ F_{a\ I} + C'_k\ (F_{k\ II} - F_{a\ I}) \tag{16}$$

where C_1 equals mean velocity in the pipe line before the multiplier. The velocities C'_e and C'_k are defined on figure 3a.

Modifying equation (16) by equation (12) gives

$$C_1\ n\ m_{II} = C_{aI} + (n - 1)\ C'_k \tag{16'}$$

Applying the Bernoulli equation successively to the flow through venturi I and the flow between venturis I and II yields

$$P_{eI} + \gamma \frac{C_{e\ I}^2}{2g} = P_{a\ I} + (1 + \xi_I) \frac{C_{a\ I}^2}{2g} \tag{17}$$

$$P'_e + \gamma \frac{C'^2_e}{2g} = P'_k + (1 + \xi') \frac{C'^2_k}{2g} \tag{18}$$

Assume that the total pressures $P_{e\ I} + \gamma \frac{C_{e\ I}^2}{2g}$ and $P'_e + \gamma \frac{C'^2_e}{2g}$ are equal (the assumption $P_{e_I} = P'_e$ gives approximately the same final result) and that by analogy with a parallel-wall channel $P_{aI} = P'_k$. Equations (17) and (18) furnish the following relationships:

$$(1 + \xi_I)\ C_{a\ I}^2 = (1 + \xi')\ C'^2_k$$

$$C_k = \varphi\ C_{a\ I} \tag{19}$$

$$\varphi = \sqrt{\frac{1 + \xi_I}{1 + \xi'}} \tag{20}$$

Since ξ_I will always be considerably greater than ξ', then $\varphi > 1.0$; therefore, the velocity $C_{a\ I}$ will always be less than the velocity C'_k. Transforming equation (16') by equation (19) gives

$$C_{a\ I} = \frac{\alpha}{(n - 1)\ \varphi + 1}\ m_{II}\ C_1 \tag{21}$$

The velocity in the minimum section of venturi I will be $C_{k\ I} = m_I\ C_{a\ I}$. Substituting this in equation (21):

$$C_{k\ I} = \frac{n}{(n - 1)\ \varphi + 1}\ m_I\ m_{II}\ C_1 \tag{22}$$

With arrangement of the pressure-augmenting device according to case 2 (fig. 1b), the pressure difference is:

$$\Delta P' = \left(P_1 + \gamma_1 \frac{C_1^2}{2g}\right) - P_{k\ I} = (1 + \zeta_e)\ \gamma_1 \frac{C_{k\ I}^2}{2g}$$

wherein ζ_e equals loss coefficient of the convergent section of venturi I.[1]

Substituting $C_{k\ I}$ in the last equation by its value according to equation (22) gives

$$\Delta P' = (1 + \zeta_e)\left[\frac{1}{(n-1)\varphi + 1} m_I\, n_{II}\right]^2 \gamma_1 \frac{C_1^2}{2g}$$

Consequently, the coefficient of augmentation for an arrangement according to case 2 is:

$$M_o = (1 + \zeta_e)\left[\frac{n}{(n - 1)\,\varphi + 1} m_I\, n_{II}\right]^2 \qquad (23)$$

and the residual loss in the pressure multiplier can be expressed as follows:

$$\delta P = \zeta\Delta P' = \zeta M_o\, \gamma_1 \frac{C_1^2}{2g} \qquad (24)$$

To find the loss coefficient ζ add the partial losses in each section of the multiplier and relate them to the discharge velocity $C_{a\ II} = C_1$ and to 1 cubic meter per second discharge volume: $Q_1 = C_1 \cdot F$ through the whole multiplier.

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[1] It may be assumed that $\zeta_e = 0$; more accurately

$$\zeta_e = \frac{\lambda}{8 \tan \frac{\theta}{2}} \frac{m_I^2 - 1}{m_I^2}$$

where λ equals frictional coefficient of the length and θ equals apex angle of the convergent length.

The resistance in the convergent lengths of both venturis can be neglected. Consequently, the residual pressure loss will be equal to the sum of the losses in the divergent lengths of venturi I:

$$\zeta_I(m_1^2 - 1) \frac{Q_I}{Q_1} \gamma_1 \frac{C_a^2{}_I}{2g}$$

and of venturi II:

$$\psi\zeta_{II}(m^2{}_{II} - 1)\gamma_1 \frac{C_1^2}{2g}$$

where $Q_I = C_{e\ I} F_{e\ I} = C_{a\ I} F_{a\ I}$ equals volume of flow through venturi I; and $\psi > 1$ equals a coefficient representing the loss through turbulence at the discharge from venturi I.

By applying equations (12) and (21)[1]

$$\delta P = \left\{\psi\zeta_{II}(m^2{}_{II} - 1) + \zeta_I(m^2{}_I - 1) \frac{n^2\ m^2{}_{II}}{[(n - 1)\varphi + 1)]^3}\right\}\gamma_1 \frac{C_1^2}{2g}$$

By combining this with equation (24), the loss coefficient of a double venturi multiplier is obtained as:

$$\zeta = \frac{1}{M_0}\left\{\psi\zeta_{II}(m^2{}_{II} - 1) + \zeta_I(m^2{}_I - 1) \frac{n^2\ m^2{}_{II}}{[(n - 1)\varphi + 1]^3}\right\} \qquad (25)$$

Now investigate individual cases:

(a) <u>Single venturi tube</u>.- The venturi tube (fig. 4) is commonly used for measuring flow velocities in a section considerably larger than the tube itself. The properties of this tube as a dynamic-pressure-augmenting device will now be examined.

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[1]The loss coefficients ζ_I and ζ_{II} can be found by the expression

$$\zeta = \frac{\lambda}{8\ \tan\frac{\theta}{2}} + \frac{m - 1}{m + 1} \sin\theta$$

where λ coefficient of friction of the length equals f(Re); and θ equals apex angle of the divergent length.

By neglecting the resistance in the convergent length of the tube - that is, assuming $\zeta_e = 0$ in equation (23), and bearing in mind that for the particular case $m_{II} = 1$ and $n \approx \infty$ - the coefficient of augmentation becomes:

$$M_o = \frac{m^2_I}{\varphi^2}$$

By disregarding the losses in the flow outside the tube, in equation (20) - that is, assuming $\xi' = 0$ - and, since $\xi_I = \zeta_I(m^2_I - 1)$, $\varphi = \sqrt{1 + \zeta_I(m^2_I - 1)}$, the coefficient of augmentation is finally expressed by:

$$M_o = \frac{m^2_I}{1 + \zeta_I\ (m^2_I - 1)} \tag{23'}$$

If the section of the venturi tube is sufficiently small compared with the flow section, the residual losses caused by the tube are negligible; consequently, the venturi tube is an ideal pressure-augmenting device. The determination of the pressure-raising capacity of the tube is therefore of eminent importance. Figure 5 shows curves of M_o and $\varphi = \frac{C_1}{C_{a\ I}}$ as functions of m_I for $\zeta_I = 0.15$. It will be seen that M_o at first increases abruptly with increasing m_I, and reaches a value of ≈ 5 for $m_I = 4$. Thence, M_o increases very slowly, and at $m_I = \infty$, attains only a value of 6.65. It has been found by experiment that at $M_I = 8.25$, only the most efficient venturi tubes of Navy and Zahm (reference 2) types, have a coefficient of augmentation of $M_o = 6.38$. Consequently, in spite of the economy of their use, venturi tubes are only practicable at very low values of M_o. Attention must be drawn to the fact that with no loss in the venturi - that is, if $\zeta_I = 0$ - then $\varphi = 1$, and $C_{a\ I} = C_1$; hence, the theoretical volume of flow through the tube will be $Q'_I = C_1 F_{a\ I}$, and the actual volume of flow:

$$Q_I = C_{a\ I}\ F_{a\ I} = \frac{Q'_I}{\varphi}$$

or φ times less than the theoretical flow. As shown by figure 5, the losses increase with increasing m_I, and thus also φ, causing the volume of the flow through the tube to diminish; the last circumstance explaining the considerable decrease in the actual value of M_o, compared with the theoretical value of $M'_o = m^2_I$.

It is now possible to define the principle of action of a pressure multiplier; which consists in the fact that, when the discharge aperture of the venturi is placed in a constricted zone of the flow (e.g., in the throat of a second, larger venturi), where the velocity is considerably increased but the pressure less than in the entry section of the tube, an artificial "draught" is created (by the pressure difference between entry and discharge of the small tube, and by the ejector effect of the velocity C'_k in the throat of the second venturi) which considerably increases the volume of flow and the velocity through the small venturi, and allows a high coefficient of augmentation.

(b) Rateau-type, twin-venturi pressure multiplier.
Firstly, it must be pointed out that the diameter $D_{k\ I}$ of the throat of the smaller venturi should not be too small, in order not to increase the coefficient of loss ζ_I.

If

$$a = \frac{D}{D_{k\ I}}; \quad \text{that is,} \quad a^2 = \frac{F}{F_{k\ I}}$$

then by equation (13)

$$n = \frac{a^2}{m_I\ m_{II}} \tag{26}$$

For φ, as in the preceding case,

$$\varphi = \sqrt{1 + \zeta_I(m^2_I - 1)} \tag{27}$$

hence, if $\zeta_e = 0$:

$$M_o = \frac{a^4}{\left[\left(\frac{a^2}{m_I\ m_{II}} \mp 1\right) \varphi + 1\right]^2} \tag{23"}$$

The coefficient of loss ξ is then by equation (25)

$$\xi = \frac{1}{M_o}\left\{\psi\zeta_{II}(m^2_{II} - 1)\zeta_I(m^2_I - 1)\frac{a^4}{m^2_I\left[\left(\frac{a^2}{m_I\, m_{II}} - 1\right)\varphi + 1\right]^3}\right\} \quad (25')$$

To determine the total length L of a multiplier, consider first the length l of the divergent cone with an apex angle of θ:

$$l = \frac{D_a - D_k}{2 \tan \frac{\theta}{2}} = \frac{D_a}{2 \tan \frac{\theta}{2}}\left(1 - \frac{1}{\sqrt{m}}\right) \quad (28)$$

Then, the length L will be for $\theta_I = \theta_{II} = \theta$, and by neglecting the short entry length of the smaller venturi tube:

$$L = \frac{D}{2 \tan \frac{\theta}{2}}\left[1 - \frac{1}{\sqrt{m_{II}}} + \frac{1}{a}(\sqrt{m_I} - 1)\right] \quad (29)$$

To assist in determining the influence of individual factors, figure 6 reproduces curves of M_I, m_{II} and ξ plotted against M_o, for the following conditions: $D = 800$ millimeters, $D_{k\,I} = 44$ millimeters, $\zeta_I = 0.1$, $\xi_{II} = 0.175$, and $\psi = 1.0$. It will be seen that for any value of M_o the loss coefficient ξ is inversely proportional to m_I, and consequently to m_{II}. For a value of $m_I = 3$ to 6, the residual pressure loss δP is only 2 to 4 percent of the pressure difference $\Delta P'$, which shows the high efficiency of this type of pressure multiplier.

<u>(c) Rateau triple-venturi pressure multiplier.</u>- Only the equations will be given for this case. By introducing the additional factors:

$$m_{III} = \frac{F_{a\,III}}{F_{k\,III}} \quad \text{and} \quad n^* = \frac{F_{k\,III}}{F_{a\,III}}$$

and assuming $\zeta_e = 0$,

$$M_o = \left[\frac{n}{(n-1)\varphi + 1}\,\frac{n^*}{(n^*-1)\varphi^* + 1}\, m_I\, m_{II}\, m_{III}\right]^2 \qquad (30)$$

where φ is calculated by equation (27) and φ^* is determined by the following expression:

$$\varphi^* = \sqrt{1 + \psi\zeta_{II}(m^2_{II} - 1) + \zeta_I(m^2_I - 1)\,\frac{n^2\, m^2_{II}}{[(n-1)\varphi + 1]^3}} \qquad (31)$$

By neglecting the insignificant losses in the smaller venturi I, the loss coefficient of the multiplier is determined by the expression:

$$\zeta = \frac{1}{M_o}\left\{\varphi\zeta_{III}(m^2_{III} - 1) + \zeta_{II}(m^2_{II} - 1)\,\frac{n^{*2}\, m^2_{III}}{[(n^*-1)\varphi^* + 1]^3}\right\} \qquad (32)$$

<u>(d) The author's multiplier.</u>- In this case (see fig. 7):

$$m_{II} = \frac{1}{\mu\left(\frac{d}{D}\right)^2} \qquad (33)$$

where μ is obtained by equation (7). The values of n, φ, and M_o are found as before, from equations (26), (27), and (23').

The residual losses in the flow behind the orifice equal:

$$\psi(m_{II} - 1)^2\,\gamma_1\,\frac{C^2_I}{2g}$$

and the loss coefficient therefore:

$$\zeta = \frac{1}{M_o}\left\{(m_{II} - 1)^2 + \zeta_I(m^2_I - 1)\,\frac{a^4}{m^2_I\left[\left(\frac{a^3}{m_I\, m_{II}} - 1\right)\varphi + 1\right]^3}\right\} \qquad (34)$$

To assist in determining the influence of individual factors, curves of d/D, m_I, and ξ are shown in figure 8, (the curves in figs. 6 and 8 have been plotted by S. M. Rothstein) plotted against values of M_o, for the conditions: $D = 800$ millimeters, $D_{k\ I} = 44$ millimeters, $\zeta_I = 0.1$, and $\psi = 1.0$.

It will be seen that at any value of M_o the residual loss coefficient is inversely proportional to m_I, and thus to d/D.

For $m_I = 3$ to 6, the corresponding residual loss is about 3 to 6 percent of the pressure difference $\Delta P'$.

II. SUCTION OR DISCHARGE GOVERNING

It is now proposed to compare the efficiency of constant-volume governing (regulation of output) with the pressure-augmenting device fitted in the suction ("suction governing") or the discharge ("discharge governing") of the compressor.

1. Coefficient of Variation of Output

The controlling force on the governor membrane equals q equals $f\ \Delta P'$, wherein f is the area of the membrane. By equation (5) the expression for the controlling force becomes:

$$q = A \frac{G^2 v}{\sigma^2} \tag{35}$$

where

$$A = M_o \frac{f}{2\,g\,F^2}$$

and

$F = \frac{\pi}{4} D^2$ cross-sectional area of the pipe before the augmentor

σ compressibility factor in the equation for the discharge volume[1]

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[1]See footnote on p. 17.

By neglecting the slight variation in the value of σ, the controlling force becomes directly proportionate to $G^2 v_1$ - G being the mass flow in kilograms per second and v_1 the specific volume of the flow before the pressure augmentor. The fact that the magnitude of the controlling force is dependent on the specific volume as well as on the mass flow also explains why the arrangement of the augmentor in the suction or the discharge, differently affects the controlling force.

To further investigate this circumstance, examine the lines of equal controlling force in the P_d - Q diagram (P_d being the final gas pressure and Q the volume flow in cubic meters per second proportional to G at constant P_s and t_s - that is, $Q = G v_s$). As will be shown further on, a governor working under ideal conditions will operate at q = constant. Then, if the pressure-augmenting device is situated on the suction side, where P_s and t_s vary only inconsiderably with atmospheric conditions - the specific volume can obviously be assumed constant throughout the governing process - that is, $v_s = v_1$. Consequently, the curves of $q = A \dfrac{G^2 v_s}{\sigma^2}$ = constant in the P_d - Q diagram will form vertical straight lines agreeing with G = constant and Q = constant. (Furthermore, if q = constant - i.e., $\Delta P'$ = constant - then σ = constant is also correct, since already $\Delta P'/P_s$ = constant.)

Conditions are different if the pressure-augmenting device is on the discharge side since P_d and t_d, and consequently, $v_d = v_1$ vary considerably during the governing process. Therefore, the governor no longer controls the value of G, but a value proportional to

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[1]For orifice plates (diaphragms):

$$\sigma = 1 - \frac{1}{k}\left(0.436 + 0.36\,\frac{d^4}{D^4}\right)\frac{\Delta P}{P_1}; \quad k = \text{adiabatic index}$$

For nozzles:

$$\sigma = 1 - \frac{1}{k}\left(0.8 + \frac{d^4}{D^4}\right)\frac{\Delta P}{P_1}; \quad \text{for} \quad \frac{\Delta P}{P_1} \leq 0.2$$

For all pressure multipliers:

$$\sigma = 1 - \frac{0.8}{k}\,\frac{\Delta P}{P_1}$$

$$\frac{G^2 v_d}{\sigma^2}$$

whence:

$$v_d = v_s \left(\frac{p_d}{p_s}\right)^{-\frac{1}{m}}$$

(m = polytropic coefficient of compression)

and consequently $$q = A \frac{G^2}{\sigma^2} \frac{v_s}{\left(\frac{p_d}{p_s}\right)^{\frac{1}{m}}} \qquad (36)$$

The last equation also shows that with G or Q constant, the controlling force q will decrease with increase of pressure - that is, the lines of constant controlling force are parabolic curves, as shown on figures 10 and 11. (For details, see below.)

Examine the simplest type of governing device (fig. 10), operated by controlling the running speed of the driver (e.g., a steam turbine). The principle of action of this governor is as follows: When the resistance in the pipe line (e.g., a blast-furnace) increases, the characteristic curve of the pipe line becomes displaced from position I to position II on the diagram. (See fig. 10.) Initially, owing to the insensitivity and inertia of the system, the steam valve does not open, and the power input N_a remains constant. The value of the revolution speed, however, becomes displaced from the point a along the N_a curve toward point e'. This diminishes the controlling force $q = A \frac{G^2 v_1}{\sigma^2}$, whether the pressure augmentor is fitted in the suction or the discharge; but this, in turn, causes the governor membrane to move upward, and with it the piston of the governor valve. By this means, pressure oil is permitted to enter underneath the servo-motor piston raising it, and opening the steam valve. The driving power then increases until the mass flow is restored to its original value. Then the sleeve of the governor valve rises, cutting off the steam and stopping the servo motor. When the governing process has thus been completed, a new condition of equilibrium is established, with the power and running speed increased, but the controlling force reduced $q_e > q_a$.

This is due to the fact that the increased power output N, $N_e > N_a$ corresponds to a higher position of the steam valve, and with it, of the governor-valve sleeve; which, however, is only possible if $q_e < q_a$. This form of governing is, therefore, only possible if the degree of irregularity of the controlling force (sensitivity of the governor) is a fixed value:

$$\delta_q = \frac{q_a - q_e}{q_m} \tag{37}$$

where

$$q_m = \frac{q_a + q_e}{2}$$

However, whether the pressure augmentor is on the suction or the delivery side, the real governing curve will always be found displaced to the left of the line of q_a = constant, the distance increasing with the value of δq. (If a cut-out (lever 7) is not provided, and the governor valve sleeve is stationary, a new condition of equilibrium can only be obtained with the piston of the governor valve in the original position - i.e., when $q_e = q_a = q$ = constant. A governor of this type would be capable of producing the "ideal" governing curve - i.e., a line of q = constant and would possess zero irregularity, $\delta_q = 0$.) Consequently, with an augmentor on the suction side, the sensitivity of governing will be about half the value of the degree of irregularity of the controlling force:

$$\sigma_Q = \frac{\theta_e - \theta_a}{\theta_m} = \frac{G_e - G_a}{G_m} \approx -\frac{1}{2}\delta_q \tag{38}$$

A different relationship obtains for an augmentor fitted on the discharge side.

It is consequently possible by suitable selection of the degree of irregularity of the controlling force q to obtain even zero degree of irregularity of the governing G or Q and thus an almost vertical governing curve. In general, the greater the range of q, the less the degree of irregularity of Q; thus, with a sufficiently large value of δ_q, the value of δ_Q may even become negative.

It is obviously very desirable, therefore, to increase the degree of irregularity of the controlling force, as contributing to the sensitivity, stability, and speed of action of the governing process. However, if the pressure-augmenting device is on the suction side, an increase in the value of δ_q simultaneously leads to an increase in δ_Q, which is, of course, undesirable. Consequently, one advantage of fitting the pressure augmentor on the discharge side, is the possibility of obtaining a very small degree of irregularity in the governing process coupled with a sufficiently high range of controlling force.

If the output G_o or Q_o is to be controlled within a pressure range of P_{d_1} to P_{d_2} (fig. 11), the line of q_o = constant passing through a point with the coordinates G_o or Q_o and $P_{d_o} = 1/2(P_{d_1} + P_{d_2})$, is represented by the following equations:

$$A \frac{G^2}{\sigma^2} \frac{v_s}{\left(\frac{P_d}{P_s}\right)^{\frac{1}{m}}} = A \frac{G^2{}_o}{\sigma^2{}_o} \frac{v_s}{\left(\frac{P_{d_o}}{P_s}\right)^{\frac{1}{m_o}}}$$

or

$$\frac{\Delta G}{G_o} = \frac{\Delta G}{Q_o} = \frac{\sigma \left(\frac{P_d}{P_s}\right)^{\frac{1}{2m}}}{\sigma_o\left(\frac{P_{d_o}}{P_s}\right)^{\frac{1}{2m_o}}} - 1 \tag{39}$$

where

$$m = \frac{k\,\eta_{pol}}{(k - 1)\eta_{pol} + 1} \tag{40}$$

η_{pol} is polytropic efficiency; k is adiabatic coefficient

and

$$\Delta G = G - G_o; \quad \Delta Q = Q - Q_o{}^1$$

---

[1]To construct the line of q = constant, determine:

$$\Delta P_o = \Delta P'_o\,\sigma^2{}_o = M_o \frac{G^2{}_o}{2g\ F^2} \frac{v_s}{\left(\frac{P_{d_o}}{P_s}\right)^{\frac{1}{m_o}}}$$

(Continued on p. 21)

In the first approximation the curve of q_o = constant is a straight line, for which:

$$\frac{\Delta Q_1}{Q_o} \sim \frac{\Delta Q_2}{Q_o} \sim \frac{\Delta P_d}{2m_o\ P_{d_o}} \qquad (41)$$

where

$$\Delta P_d = P_{d_2} - P_{d_o} = P_{d_o} - P_{d_1}$$

If the governing curve is to have its origin at point 1 and terminate at point 2 (fig. 11), corresponding to zero mean irregularity of Q or G - that is, to $\delta_Q = 0$ - it will be necessary for the degree of irregularity of the governing force to have a value of:

$$\delta_q = \frac{q_1 - q_2}{q_o} \qquad (42)$$

A first approximation for δ_q can be obtained by the following equation:

$$\delta_q \sim 4\ \frac{\Delta Q_1}{Q_o}\ 100 = \frac{2}{m_o}\left(\frac{\Delta P_d}{P_{d_o}}\right)\ 100 \text{ percent} \qquad (43)$$

From this it follows, for instance, that if it is desired to control Q within a degree of irregularity of $\delta_Q = 0$ and within pressure limits of ±10 percent of P_{d_o}, it is necessary for the controlling force to have a degree of irregularity of δ_q = 10 to 13 percent (m_o between 1.56 and 2.0)

(Continued from p. 20) and thence, σ_o and $\Delta P'_o = \frac{\Delta P_o}{\sigma^2{}_o}$. Further, values of η_{pol} (with $Q = Q_o$), m, and σ $\left(\text{by } \frac{\Delta P_o}{P_d}\right)$ corresponding to a range of values of P_d between P_{d_1} and P_{d_2}, must be determined. The required values of ΔG and G, or ΔQ and Q, will be found by equation (39).

2. Steadiness of Governing

Without going into details, if the pressure augmentor is fitted in the discharge, the dynamics of governing are affected by the appearance of a negative term in one of the factors of steadiness. Since, however, the governing process of a centrifugal compressor is steady in all cases (reference 3) (so long as the compressor is operating in the steady region of the characteristic), this circumstance will hardly be material.

3. Power Lost in Governing

Owing to the necessary high pressure differences ΔP, (reference 3), the residual pressure losses and accompanying power losses are of material significance in the problem of governing.

If, on account of residual pressure losses in the pressure-augmenting device, the initial pressure P_s is reduced by the amount δP_s or the final pressure P_d by the amount δP_d, the compression ratio must be increased to:

$$\epsilon' = \frac{P_d}{P_s - \delta P_s} = \frac{\epsilon}{1 - \dfrac{\delta P_s}{P_s}}$$

in the first case, and:

$$\epsilon' = \frac{P_d + \delta P_d}{P_s} = \epsilon\left(1 + \frac{\delta P_d}{\epsilon P_s}\right)$$

in the second case.

The second expression is connected with the excess power coefficient $\dfrac{\delta N}{N} = \dfrac{N' - N}{N}$, which is:

(a) Pressure-augmentor on the suction side:[1]

---

[1]

$$\lambda = \frac{m}{m - 1} = \eta_{pol} \frac{k}{k - 1}$$

Uncooled compressor:

$$\frac{\delta N_s}{N} = \frac{\epsilon'^{\frac{1}{\lambda}} - \epsilon^{\frac{1}{\lambda}}}{\epsilon^{\frac{1}{\lambda}} - 1} = \frac{\left(1 - \frac{\delta P_s}{P_s}\right)^{-\frac{1}{\lambda}} - 1}{1 - \epsilon^{-\frac{1}{\lambda}}} \tag{44}$$

Cooled compressor:

$$\frac{\delta N_s}{N} = \frac{\lg \epsilon' - \lg \epsilon}{\lg \epsilon} = \frac{-\lg\left(1 - \frac{\delta P_s}{P_s}\right)}{\lg \epsilon} \tag{45}$$

(b) Pressure augmentor on the discharge side:

Uncooled compressor:

$$\frac{\delta N_d}{N} = \frac{\left(1 + \frac{\delta P_d}{\epsilon P_s}\right)^{\frac{1}{\lambda}} - 1}{1 - \epsilon^{-\frac{1}{\lambda}}} \tag{46}$$

Cooled compressor:

$$\frac{\delta N_d}{N} = \frac{\lg\left(1 + \frac{\delta P_d}{\epsilon P_s}\right)}{\lg \epsilon} \tag{47}$$

Sufficiently accurate values can be obtained by the following series expansions:

Uncooled compressor:

$$\frac{\delta N_s}{N} = \frac{\frac{\delta P_s}{P_s}}{1 - \epsilon^{-\frac{1}{\lambda}}} \tag{44'}$$

Cooled compressor:

$$\frac{\delta N_s}{N} = \frac{\frac{\delta P_s}{P_s}}{2.303 \lg \epsilon} \tag{45'}$$

Uncooled compressor:

$$\frac{\delta N_d}{N} = \frac{\frac{\delta P_d}{P_s}}{\epsilon\left(1 - \epsilon^{-\frac{1}{\lambda}}\right)} \tag{46'}$$

Cooled compressor:

$$\frac{\delta N_d}{N} = \frac{\frac{\delta P_d}{P_s}}{3.303\epsilon \lg \epsilon} \tag{47'}$$

These formulas indicate that the relative excess power coefficient $\frac{\delta N}{N}$ is directly proportional to the ratio $\frac{\delta P}{P_s}$ and varies inversely with the compression ratio ϵ (see also figs. 12a and 12b).

To compare the power loss in the suction with the power loss in the discharge, consider the ratio $\frac{\delta N_s}{\delta N_d}$. Since:

$$\frac{\delta N_s}{\delta N_d} = \epsilon \frac{\delta P_s}{\delta P_d} \tag{48}$$

if the residual loss δP is equal on both sides - that is, $\delta P_s = \delta P_d$ - then an augmentor in the discharge will produce a power loss ϵ times greater than the power loss on the suction side.

To compare the pressure losses δP_s and δP_d, consider the simplest form of pressure augmentor - a nozzle fitted as in case 1 (fig. 1). Since:

$$M_0 = \frac{\Delta P'}{\gamma_1 \frac{C^2{}_1}{2g}}$$

then, by equations (4) and (6), by assuming $\sigma = 1$:

$$\delta P = \left[\alpha\sqrt{\Delta P'} - \sqrt{\gamma_1 \frac{C^2_1}{2g}}\right] \tag{49}$$

Hence, with equal pressure differences $\Delta P'$, the residual pressure loss on the discharge side δP_d will be less than the corresponding loss on the suction side δP_s, since in the former case

$$\gamma_d = \gamma_s \epsilon^{\frac{1}{m}} > \gamma_s$$

Actually, therefore:

$$\frac{\delta N_s}{\delta N_d} > \epsilon$$

This is demonstrated by figure 13, showing the appropriate values of H_0 and the power losses $\delta N_s/N$ and $\delta N_d/N$, for $\Delta P' = 2000$ millimeters water and different types of pressure augmentors.

By summarizing paras. 3, 4, and 5, from the point of view of the degree of irregularity of the governing process and the magnitude of the accompanying power losses, discharge governing is more efficient than suction governing.

III. DESIGN AND SELECTION OF PRESSURE AUGMENTORS

1. Selection of the Type of Augmentor

When selecting the most suitable type of pressure-augmenting device, the following considerations should be borne in mind: 1) arrangement (suction or delivery side); 2) power losses; 3) structural length; 4) cost and simplicity of manufacture; 5) fitting, supervision; and maintenance. The selection depends entirely on which of the above factors is to be considered most important.

For compression ratios between $\epsilon = 1.5$ to 4.0 - that is, for uncooled compressors - the following indications may be used (see also fig. 13): 1) Orifice plates (diaphragms) and nozzles are unsuitable for this case since, for example, the power loss when fitted in the discharge exceeds 3 percent at $\Delta P' = 2000$ millimeters water, and 1 percent when fitted in the suction. 2) Plain venturis are unsatisfactory as requiring very high values of M. 3) Rateau twin or triple venturis can only be used on the suction; furthermore, only at low values of ϵ and sufficiently high values of $\Delta P'$. 4) The author's multiplier is recommended for use on the delivery side, since it couples a sufficiently low power loss coefficient with simplicity and compactness of construction.

It should be particularly mentioned that when governing cooled compressors (turbo-compressors) from the delivery side, orifice plates and nozzles are perfectly suitable as pressure-augmenting devices; at a ratio of $\epsilon > 6$ to 7, plain venturi tubes are also satisfactory, since the requisite coefficient of augmentation is $M < 7$. When governing turbocompressors from the suction side, the author's type of pressure multiplier is quite satisfactory.

2. Remarks on the Design of Pressure Augmentors

Orifice plates and nozzles as pressure-augmenting devices can be designed in all respects along normal lines (reference 4). If simultaneously acting as flow meters, they should be provided with piezometer rings.

In designing pressure multipliers, full use should be made of the available experience in the construction of venturi tubes.

Although the diverging length

$$l = \frac{D_a - D_k}{2 \tan \frac{\theta}{2}} = \frac{D_a}{2 \tan \frac{\theta}{2}} - \left(1 - \frac{1}{\sqrt{m}}\right) \tag{28}$$

should be as short as possible, the apex angle should never exceed 6°,- at most 7° to 8° - to avoid break away of the flow and consequent increased flow losses.

The throat diameter $D_{k\ I}$ of venturi I should preferably be more than 20 to 30 millimeters. The angle θ for the converging entry length is usually 20°. The intake section $F_{e\ I}$ should be sufficient to ensure that $C_{e\ I} = C_{1\ max} = K_c\ C_1$, where $C_{1\ max}$ is the velocity in the axis of the pipe at a sufficient distance upstream. In such case, the following expression derived from equation 22, can be used for all pressure multipliers excepting the Rateau triple-venturi type:

$$F_{e\ I} = F_{a\ I} \frac{1}{K_c} \frac{n}{(n - 1)\varphi + 1} m_{II} \tag{50}$$

But if the value of $F_{e\ I}$ is taken lower than by the above equation, there is obtained $C_{e\ I} > C_{1\ max}$ with the result that the normal pressure and velocity field in the pipe becomes distorted, with increased irregularity and additional losses. The coefficient K_c may be taken at 1.0 to 1.2. When designing a pressure multiplier of the author's type, the discharge of the venturi $F_{a\ I}$ should preferably be situated in the minimum section of the flow behind the diaphragm, which will be at about:

0.2D to 0.35D for $d/D = 0.85$

0.35D to 0.5D for $d/D = 0.75$

0.5D to 0.7D for $d/D = 0.7$

The entry section $F_{e\ I}$ should be distant at least 0.1D to 0.6D from the diaphragm.

It may be mentioned in closing, that a number of the propositions set up in this paper, still need experimental checking and development.

Translation by L. J. Goodlet.

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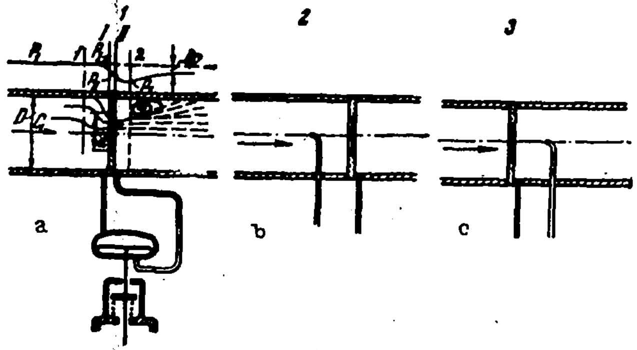
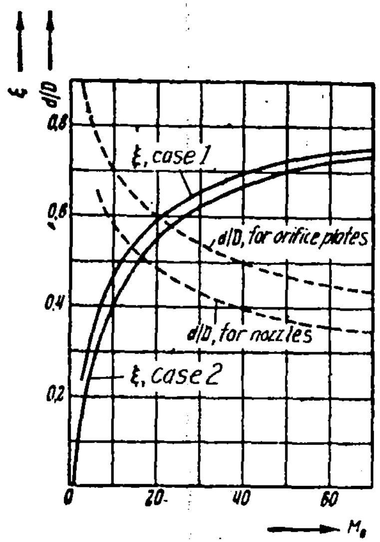


Figure 1.- Alternative arrangements of the pressure-augmenting device: (a) case 1; (b) case 2; (c) case 3.

Figure 2.- Loss coefficient ξ, and ratio d/D for orifice-plates and nozzles (d/D for augmentor after case 2).



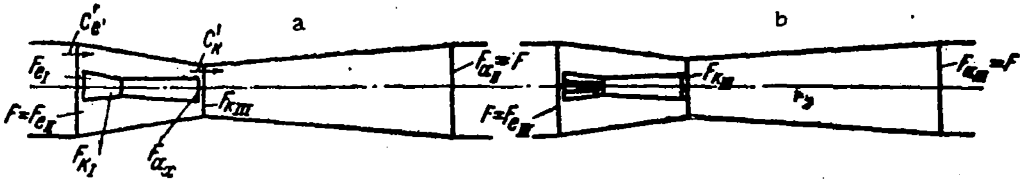


Figure 3.- Rateau pressure multipliers: (a) twin venturi; (b) triple venturi.

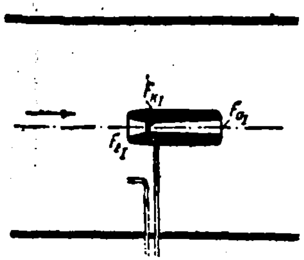
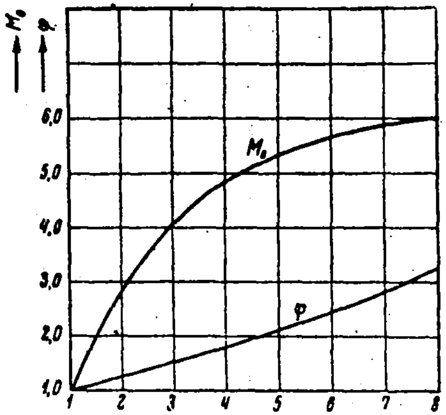


Figure 4.- Venturi tube.

Figure 5.- Relationship between values of M_0, φ, and m_I of a venturi tube.



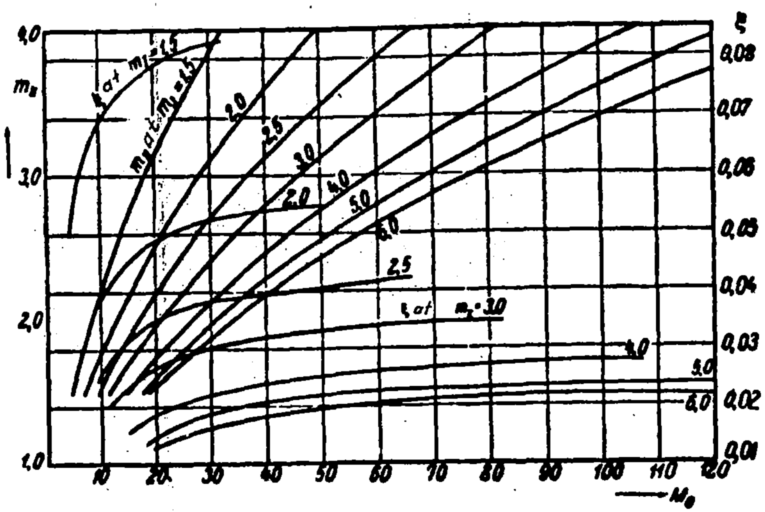


Figure 6.- Design curves for a Rateau twin-venturi multiplier.

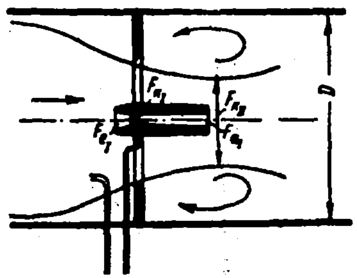


Figure 7.- The Ris multiplier.

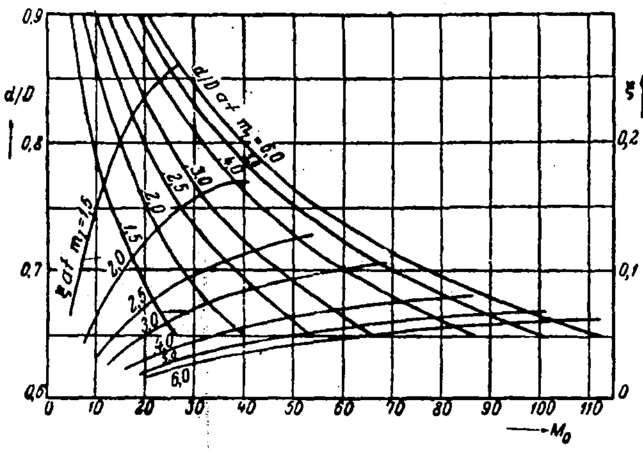


Figure 8.- Design curves for a Ris multiplier.

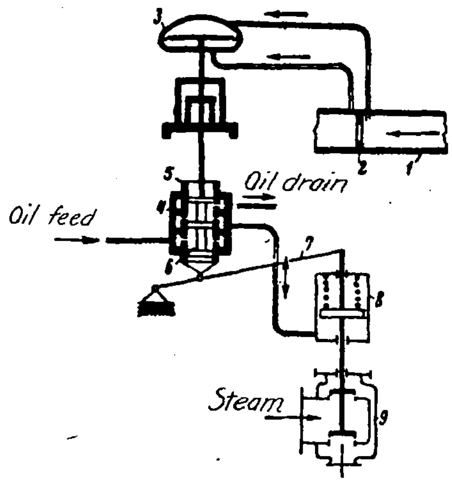


Figure 9.- Arrangment diagram of an output governor.

1. Intake and discharge pipe-line.
2. Pressure-augmenting device.
3. Output governor (controlling member).
4. Governor valve (piston valve).
5. Governor valve sleeve.
6. Governor valve spindle.
7. Cut-out lever.
8. Servo-motor.
9. Steam valve.

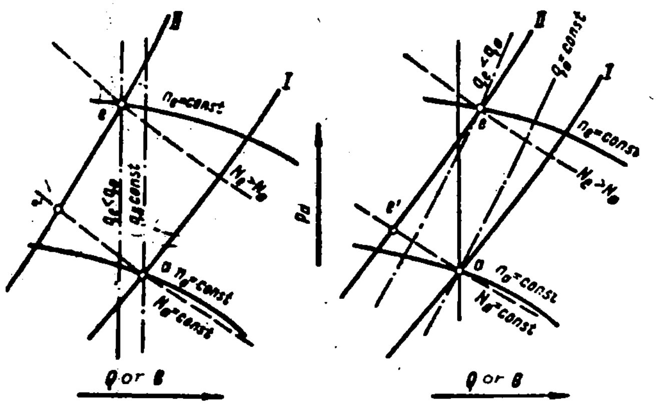


Figure 10.- Running-speed variation for suction (left) and discharge (right) governing.

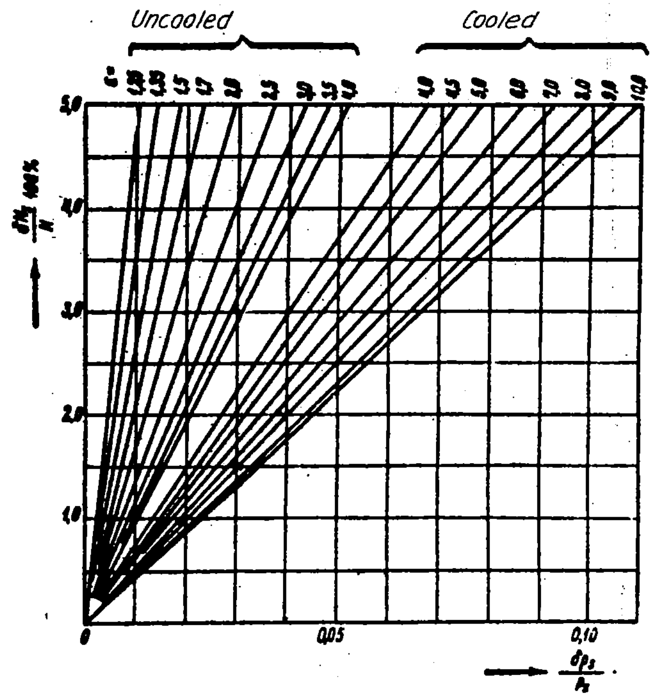


Figure 12a.- Power loss $\frac{\delta N_s}{N}$. 100 related to $\frac{\delta P_s}{P_s}$; δP_s = residual pressure loss on the suction side.

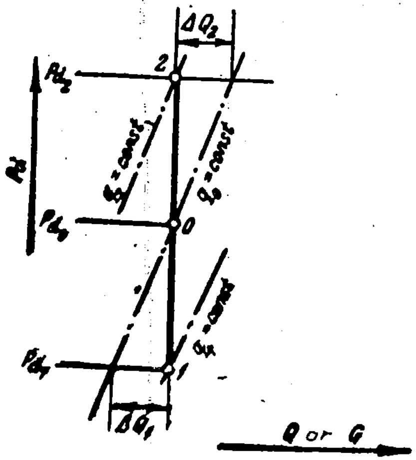


Figure 11.- Curve of q = const. and irregularity δ_q, at δ_Q = 0.

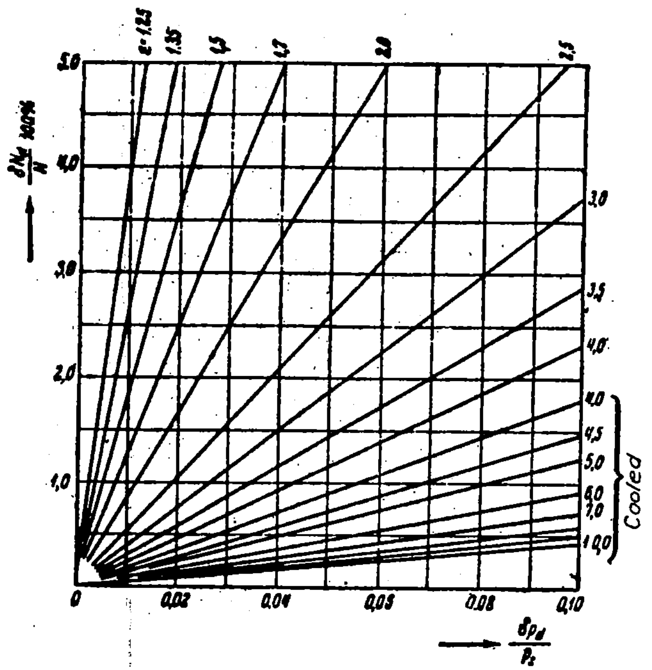


Figure 12b.- Power loss $\frac{\delta N_d}{N}$ 100 related to $\frac{\delta P_d}{P_s}$; δP_d = residual pressure loss on the discharge side

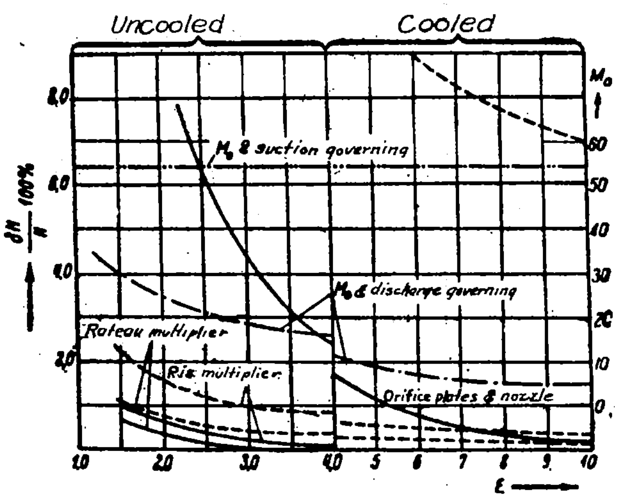


Figure 13.- Power losses $\frac{\delta N}{N}$ · 100 produced by different types of pressure augmentor fitted in the suction or the discharge, with the requisite coefficients of augmentation M_o (assumed $\Delta P'$ = 2000 mm of water). Full-line represents discharge governing and dotted line represents suction governing.